

Show your work on this exam. CIRCLE YOUR ANSWERS. Be neat.

1. Use three iterations of Newton's method to find  $x$  such that  $x^3+x^2=11$ . Start with  $x_0=2$ . Show all the intermediate results.

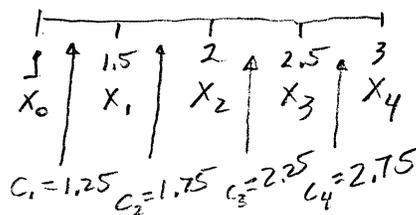
$$f(x) = x^3 + x^2 - 11 = 0$$

$$f'(x) = 3x^2 + 2x$$

$n$	$x_n$	$f(x_n)$	$f'(x_n)$	$x_n - f(x_n)/f'(x_n)$
0	2	1	16	$2 - \frac{1}{16} = \frac{31}{16}$
1	$\frac{31}{16}$	$\frac{111}{4096} = .0271$	$\frac{3875}{256} = 15.1367$	1.935709677
2	1.935709677	.00002183	15.11233522	1.935708233

2. Use the Midpoint Rule with  $n=4$  to approximate

$$\int_1^3 x^3 dx$$



$$\Delta x = \frac{b-a}{n} = \frac{3-1}{4} = \frac{1}{2}$$

$$\int_1^3 x^3 dx \approx f(c_1)\Delta x + f(c_2)\Delta x + f(c_3)\Delta x + f(c_4)\Delta x$$

$$= 1.25^3 \frac{1}{2} + 1.75^3 \frac{1}{2} + 2.25^3 \frac{1}{2} + 2.75^3 \frac{1}{2} = 19.75$$

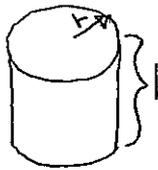
4. State the definition for the indefinite integral.

The most general antiderivative

5. Determine the dimensions (height and radius) of a cylindrical soup can that contains a volume of 10 cubic inches and uses a minimum amount of metal.

Amount of metal = area top + area bottom + area cylinder

$$= \pi r^2 + \pi r^2 + 2\pi rh = 2\pi r^2 + 2\pi rh = S$$



Volume of can =  $\pi r^2 h = 10$

so  $h = \frac{10}{\pi r^2}$

$$2\pi r^2 + 2\pi r \left( \frac{10}{\pi r^2} \right) = S$$

$$S = 2\pi r^2 + \frac{20}{r}$$

$$\frac{dS}{dr} = 4\pi r - \frac{20}{r^2} = 0$$

$$r^3 = \frac{5}{\pi} \Rightarrow r = \sqrt[3]{\frac{5}{\pi}} \approx 1.1675$$

$$h = \frac{10}{\pi r^2} \approx 2.335$$

7. practice:  $\int x^n dx = \frac{x^{n+1}}{n+1} + c$  for  $n \neq -1$

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$$\int \sin(x) dx = -\cos(x) + c$$

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$$\int \cos(x) dx = \sin(x) + c$$

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$$\frac{d}{dx} x^n = n x^{n-1}$$

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$$\frac{d}{d\theta} \sin(\theta) = \cos(\theta)$$

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$$\frac{d}{dt} \cos(t) = -\sin(t)$$

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$$\frac{d}{dy} \tan(y) = \sec^2(y)$$

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$$\frac{d}{dx} \sec(x) = \sec(x) \tan(x)$$

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$$d(x^n) = n x^{n-1} dx$$

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$$d(\sin(\theta)) = \cos(\theta) d\theta$$

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$$d(\cos(t)) = -\sin(t) dt$$

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$$d(\tan(y)) = \sec^2(y) dy$$

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$$d(\sec(x)) = \sec(x) \tan(x) dx$$